Exercise 52

Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

$$\frac{d^{35}}{dx^{35}}(x\sin x)$$

Solution

Start calculating derivatives.

$$\frac{d^1}{dx^1}(x\sin x) = \sin x + x\cos x$$

$$\frac{d^2}{dx^2}(x\sin x) = \cos x + \cos x - x\sin x = 2\cos x - x\sin x$$

$$\frac{d^3}{dx^3}(x\sin x) = -2\sin x - \sin x - x\cos x = -3\sin x - x\cos x$$

$$\frac{d^4}{dx^4}(x\sin x) = -3\cos x - \cos x + x\sin x = -4\cos x + x\sin x$$

$$\frac{d^5}{dx^5}(x\sin x) = 4\sin x + \sin x + x\cos x = 5\sin x + x\cos x$$

$$\frac{d^6}{dx^6}(x\sin x) = 5\cos x + \cos x - x\sin x = 6\cos x - x\sin x$$

$$\frac{d^7}{dx^7}(x\sin x) = -6\sin x - \sin x - x\cos x = -7\sin x - x\cos x$$

Notice that every four derivatives, the function repeats itself aside from the first coefficient. Since $35 = 3 + 4(8)$,

$$\frac{d^{35}}{dx^{35}}(x\sin x) = -35\sin x - x\cos x.$$